Analysis of Uniform Magnetic Field Problem by Edge Elements – Effects of Reduced Integration and High Order Elements –

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Abstract — Uniform magnetic field problem is widely appeared in the validation at developing finite element software, and so on. However, it is difficult to realize the uniform magnetic field in distorted hexahedral isoparametric elements. Then, authors introduced the subparametric elements. Furthermore, the reduced integration of Gauss-Legendre quadrature is applied to the first-order isoparametric hexahedral edge elements to improve the uniformity. In this paper, we discussed the effectiveness of various techniques to realize the uniform magnetic field.

I. INTRODUCTION

It is important to analyze the uniform magnetic field problem for the validation at developing software using the finite element method. The accurate computation of uniform field is strongly required in, for example, the imaged domain of MRI equipment. Therefore, some higher-order elements have been proposed [1] – [3]. However, the discrepancy of the discretization order between coordinates and magnetic vector potentials in the isoparametric hexahedral elements unexpectedly makes numerical error in the uniform magnetic field problem.

Then, one of the authors introduced the subparametric element discretized by the second-order edge-based shape function for the magnetic vector potentials and the first-order nodal shape function for coordinates [4]. On the other hand, a reduced integration technique [5] is successfully applied to the structural analysis of plates and shells. However, the paper regarding the application of reduced integration to the uniform problem is not reported yet.

This paper examined the performance of higher-order elements and the reduced integration in the uniform field problem, and these effective characteristics are discussed.

II. DISCRETIZED ELEMENT

A. Second-Order Edge Hexahedral Element

When the vector shape function $\boldsymbol{N}_{ke}$ is defined by the second-order hexahedral edge element, one of the serendipity type of edge-based shape function can be expressed as follows:

$$\boldsymbol{N}_{ke} = \frac{1}{8}(1+\eta_{ke}\eta)(1+\zeta_{ke}\zeta)(4\xi_{ke}\xi+\eta_{ke}\eta+\zeta_{ke}\zeta-1)\nabla\xi \,, \quad (1)$$

$$\boldsymbol{N}_{ke} = \frac{1}{4}(1+\zeta_{ke}\zeta)(1-\eta^2)\nabla\xi \,, \quad (2)$$

where ξ, η, ζ are the local coordinates, respectively. (1) is for edges on element's edges, and (2) for edges on the element's surface. While the term $\nabla\xi$ in the second-order subparametric element is calculated by using the first-order nodal shape function, it is defined by using the second-order shape function in the isoparametric elements.

B. Reduced Integration

When the isoparametric hexahedral element with the first-order is applied to the magnetic field analysis, integral points of 2×2×2 or 3×3×3 is generally adopted. However, it is difficult to realize the uniform magnetic field in distorted elements. Then, a reduced integration technique [2] is introduced in the first-order isoparametric hexahedral elements. The number of integration points of Gauss-Legendre quadrature is simply decreased to a single.

III. NUMERICAL ANALYSIS

A. Analyzed Model

Fig. 1 shows the two analysis models. The model (a) shows the simple cube, in which the flux density in the z-direction is defined as 1 T. The model (b) shows the spherical magnetic material model. The theoretical flux density in the spherical body is 2.994 T when its relative permeability μ_r is set at 1000. However, this theoretical value is not rigorously estimated because of not taking magnetic field condition of spherical body into account. The uniform field is realized by the boundary condition of vector potentials. The number of the first-order hexahedral elements is eight times as large as that of the second-order elements in both models.

B. Numerical Results

Fig. 2 shows the distributions of flux density in the cube model. The first-order isoparametric hexahedron (iso-

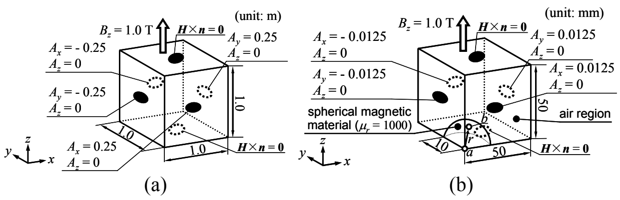

Fig. 1. Analyzed model : (a) cube mode. (b) spherical magnetic material model.

hexa) with 2×2×2 integral points cannot give the uniform field as shown in (b). On the other hand, the uniformity is improved by the reduced integration as shown in (c).

TABLE I shows the analysis results of cube model. The number of ICCG iterations for the first-order iso-hexa with the reduced integration is three times as large as that for the first-order iso-hexa with the conventional 2×2×2 integration. In this model, the first-order tetrahedral element (iso-tetra), iso-hexa with the reduced integration and the subparametric hexahedron (sub-hexa) give the enough uniformity.

Fig. 3 shows the distributions of flux density on the surfaces of spherical body. The reduced integration applied to the 1st iso-hexa slightly improves the uniformity. While the sub-hexa generally realizes the uniform field, the second-order isoparametric element (2nd iso-hexa) fails to model the uniform field. However, the accuracy of flux density of second iso-hexa may be higher in the inner domain of spherical body as shown in Fig. 4 (a) because the magnetic reluctivity is decreased by the second-order shape approximation of the spherical surfaces. The horizontal axis of Fig. 4 (a) shows the distance of line *a*-*b* in Fig. 1 (b).

Fig. 4 (b) shows the convergence characteristics of magnetic energy in the spherical body. The distribution of 2nd iso-hexa is most close to the theoretical value because of the well-modelling of the spherical shape. The discrepancy between 1st iso-hexa (1) with reduced integration and 2nd sub-hexa is little or nothing, when the mesh size is small. The characteristic of 1st iso-hexa (1) is generally superior to that of 1st iso-hexa (2×2×2).

TABLE II shows the analysis results of spherical magnetic material model. The CPU time of the second-order element is about twice as long as that of the first-order hexahedral elements. The convergence of ICCG in the second-order elements gets worse against the first-order elements because of the increase of nonzero entries.

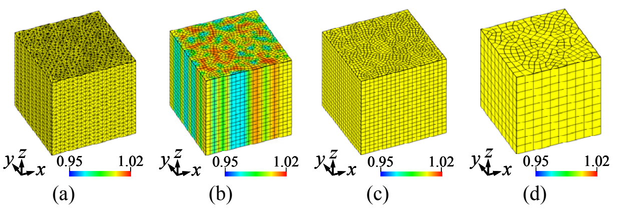

Fig. 2. Distributions of flux density in cube model : (a) 1st iso-tetra. (b) 1st iso-hexa (2×2×2). (c) 1st iso-hexa with reduced integration. (d) 2nd sub-hexa (3×3×3).

IV. REFERENCES

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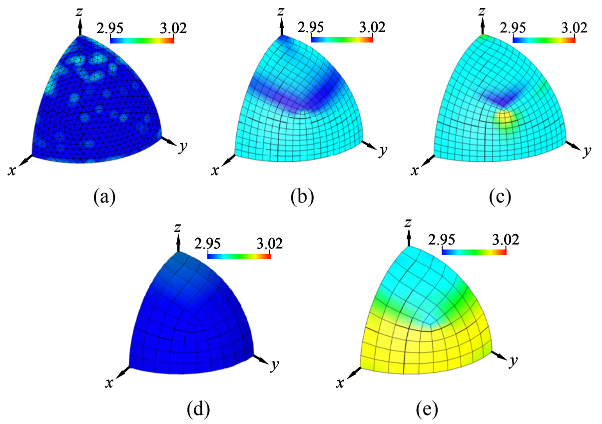

Fig. 3. Distributions of flux density in spherical magnetic material model : (a) 1st iso-tetra. (b) 1st iso-hexa (2×2×2). (c) 1st iso-hexa with reduced integration. (d) 2nd iso-hexa (3×3×3). (e) 2nd sub-hexa (3×3×3).

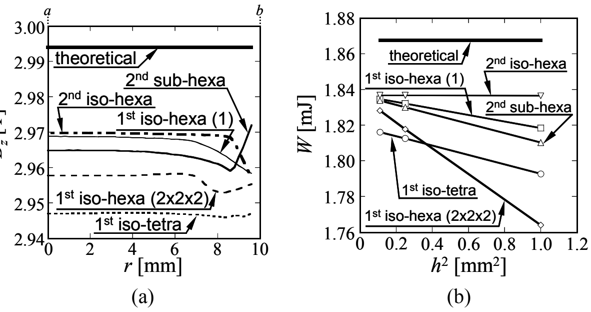

Fig. 4. Some characteristics in spherical magnetic material model : (a) B_z in inner spherical body on line *a*-*b*. (b) convergence of energy in spherical material against element size (square of element width).

TABLE I
ANALYSIS RESULTS OF CUBE MODEL

element type	order of *A*	NoE	integ. points	DoF	nonzero	ICCG ite.	CPU time [s]	B_{min} [T]	B_{max} [T]
iso-tetra	1st	114,960	-	134,574	1,137,962	86	2.8	1.000	1.000
iso-hexa		19,160	1	57,776	932,888	131	2.6	1.000	1.000
			2x2x2			40	1.7	0.945	1.017
sub-hexa	2nd	2,440	3x3x3	29,524	1,149,050	53	3.0	1.000	1.000

TABLE II
ANALYSIS RESULTS OF SPHERICAL MAGNETIC MATERIAL MODEL

element type	order of *A*	NoE	integ. points	DoF	nonzero	ICCG ite.	CPU time [s]	B_{min} [T]	B_{max} [T]
iso-tetra	1st	96,986	-	111,204	930,021	78	2.2	2.885	3.022
iso-hexa		16,000	1	46,790	749,650	61	1.3	2.934	3.015
			2x2x2			57	1.3	2.938	2.969
	2nd	2,000	3x3x3	23,090	879,123	79	2.5	2.932	2.971
sub-hexa						79	2.4	2.953	3.007